2.2.1 Additional

Lecture Notes

KCS303 Discrete Structures and Theory of Logic

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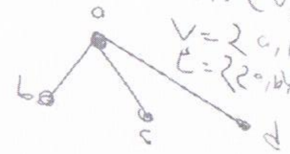
Graph

A Graph consist of a pair (V, E) where V is a non-Empty finite set of vertices and E is set of unordered pair of distinct vertices

$V \to$ vertices/points/nodes

E - Edges/lines.



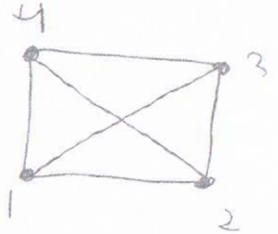
$G = (V, E)$

$V = \{a, b, c, d\}$

$E = \{\{a,b\}, \{a,c\}, \{a,d\}\}$

A graph with m vertices and n edges is called a (m, n) Graph

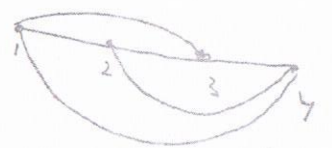
Example

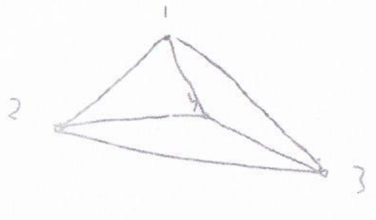


G is $(4, 6)$ graph

$V = \{1, 2, 3, 4\}$

$E = \{\{1,2\}, \{1,3\}, \{1,4\}, \{2,3\}, \{2,4\}, \{3,4\}\}$





Directed Graph

A directed graph $G = (V, E)$ when V is a nonempty finite set of vertices, and E is a set of ordered pairs of vertices

if $e = (u, v)$ the u is the starting point & v is end point

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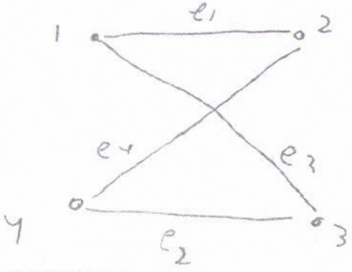
Another definition of Graph

Ex



$V = \{1, 2, 3, 4\}$

$E = \{e_1, e_2, e_3, e_4\}$

$\phi(e_1) = \{1, 2\}$
$\phi(e_2) = \{3, 4\}$
$\phi(e_3) = \{1, 3\}$
$\phi(e_4) = \{2, 4\}$

A graph $G = (V, E, \phi)$

where V = non empty finite set of vertices
E = finite set of edges.
ϕ = is a function the map the elements of E to set of unordered pairs of elements of V

Adjacent edges

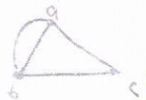
If two edges are incident on same vertex, they are called adjecent edges.

loop (self loop)

An edge from a vertex to itself is called a loop

Multiple edges / Parallel edges

when more than one edge associated with a given pair of vertices

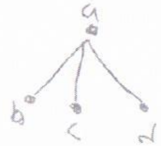


Adjacent vertices

Vertices u and v are said to be adjacent if there is an edge $e = \{u, v\}$

The edge, e, is said to be incident to node u and v

example



The vertices a and b are adjacent
The vertices b and c are non adjacent

The definition of graph does not allow multiple edges and loops.

A Graph has no loop and has no multiple edges are called a simple Graph

Multigraph

A graph which contain some multiple edges is called a Multigraph

Pseudograph

Graph that may include loops and possibly multiple edges are called a Pseudo graph

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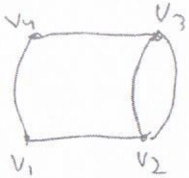
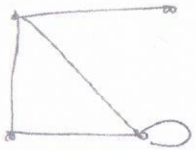


Multigraph



Pseudograph

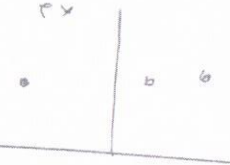
Null graph (Totally disconnected graph)

$G = (V, E)$

V = non empty finite set of vertices

E = Empty

ex



Trivial Graph

$G = (V, E)$

V = is one vertex only

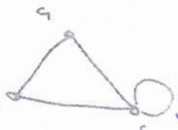
E = Empty

Degree of a vertex

The degree of a vertex, v, in a graph is the number of edges incident with v.

The degree of vertex, v, is denoted by $\deg(v)$ or $\deg_G(v)$ or $d(v)$

→ A loop contributes 2 to the degree of a vertex

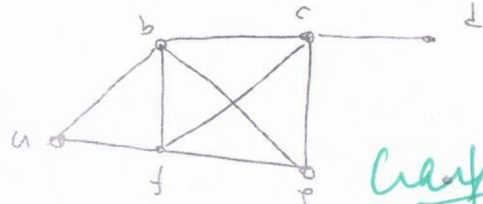


$\deg(a) = 2$
$\deg(b) = 2$
$\deg(c) = 4$

→ A vertex with degree 0 will be called as an isolated vertex

→ A vertex is pendant if it has degree one. (end point)

Find the degree of each vertex



$\deg(a) = 2$
$\deg(b) = 4$
$\deg(e) = 4$
$\deg(d) = 1$
$\deg(c) = 3$
$\deg(f) = 4$
$\deg(g) = 0$

The Handshaking theorem

let $G = (V, E)$ be an undirected graph with e edges, then

$$\sum_{v \in V} \deg(v) = 2 \cdot e$$

→ The sum of degree of vertices is twice the no of edges.

⇒ The sum of degree of vertices in an undirected graph is even

proof

∴ Every edge of G is incident with two vertices, hence every edge contributes 2 to the sum of degrees of the vertices.

Hence, $$\sum_{v \in V} \deg(v) = 2e$$

Q In any graph G, the no of vertices of odd degree is even.

Proof: Using handshaking theorem,

$$\sum_{v \in V} \deg(v) = 2.e$$

$$\sum_{\text{even}} \deg(v) + \sum_{\text{odd}} \deg(v) = 2e$$

$$\sum_{\text{odd}} \deg(v) = 2e - \sum_{\text{even}} \deg(v)$$, that is even.

$\sum_{\text{odd}} \deg(v)$ will be even if and only number of vertices of odd degree is even.

Q Show that the maximum number of edges in a simple graph with n vertices is $\frac{n(n-1)}{2}$

Using handshaking theorem

$$\sum_{i=1}^{n} \deg(v_i) = 2e$$

$$\deg(v_1) + \deg(v_2) + \text{——} + \deg(v_n) = 2e$$

maximum degree of each vertex in a simple graph can be $n-1$

$$(n-1) + (n-1) + \cdots + (n-1) = 2e$$

$$n(n-1) = 2e$$

$$e = \frac{n(n-1)}{2} \qquad \binom{n}{2}$$

Q. Let G be a graph with ten vertices. If four vertices has degree 4 and six vertices has degree 5. then find the no of edges.

Sol. Using handshaking theorem

$$\sum_{v \in V} \deg(v) = 2e$$

$$4 \times 4 + 6 \times 5 = 2e$$

$$e = 46/2 = 23$$

Q. Show that there does not exist a graph with 5 vertices with degree 1, 3, 4, 2, 3 respectively

Sol. Using handshaking theorem

$$\sum_{v \in V} \deg(v) = 2e$$

$$1 + 3 + 4 + 2 + 3 = 2e$$

$\sum_{v \in V} \deg(v)$ should be even

$e = 13/2$ that is no possible to have no of edges in fraction.

So graph does not exist

Complete Graph

A graph in which any two distinct vertices are adjacent is called a complete graph

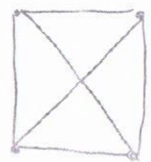
A simple graph G is said to be complete graph if every vertex in G is connected with every other vertex

A complete graph is denoted by 'K_n'

K_1

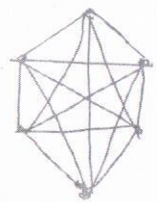
K_2

K_3



K_4

K_5



K_6

No of edges in a K_n Graph

$$= \frac{n(n-1)}{2} \quad {}^nC_2$$

Regular Graph

A graph in which all vertices are of equal degree is called a regular graph

If Graph has n vertices and is regular graph of degree r. then

G has $\frac{nr}{2}$ edges

Proof Using Handshaking theorem

$$\sum_{v \in V} \deg(v) = 2e$$

$$nr = 2e$$

$$\boxed{e = \frac{n.r}{2}}$$

If the degree of each Vertex is r then graph is called a regular graph of degree r.

→ Every Null graph is a regular graph of degree $\underline{0}$

→ The Complete graph K_n is a regular graph of degree $\underline{(n-1)}$

Q Does there exist a 4-regular graph on 6 vertices. If so construct a graph.

Using handshaking theorem

$$6 \times 4 = 2e$$

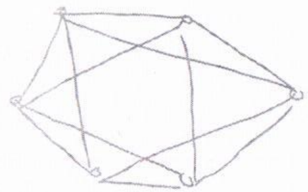
$$e = 12$$

$$\frac{6(6-1)}{2}$$

$$15$$

$12 \leq 15$

So there exist such graph



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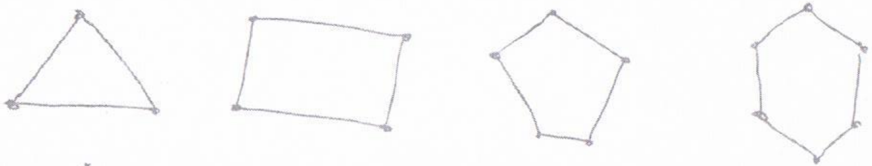
Cycle

The cycle C_n, $n \geq 3$, consist of n vertices $v_1, v_2, \ldots v_n$ and edges $\{v_1, v_2\}, \{v_2, v_3\} \ldots \{v_{n-1}, v_n\}$ and $\{v_n, v_1\}$



C_3 C_4 C_5 C_6

Wheel

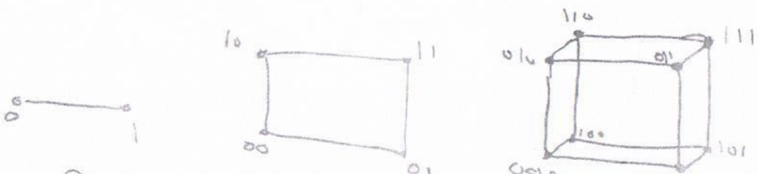
The wheel graph W_n $(n \geq 3)$ is obtained from cycle C_n by adding an additional vertex inside cycle and connecting it to every vertex in C_n

W_3 W_4 W_5

N-cube

The n-cube denoted by, Q_n, is a graph that has vertices representing the 2^n bit strings of length n.

Two vertices are adjacent if and only if the bit strings that they represent differ in exactly one bit position.



Q_1 Q_2 Q_3

n length binary no.

n-bit

vertices $= 2^n$

degree $= n$

$\sum \deg(v) = 2e$

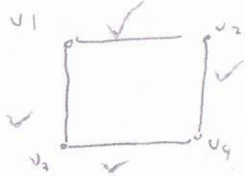
$n \cdot 2^n = 2e$

$e = \frac{n \cdot 2^n}{2}$

$\boxed{e = n \cdot 2^{n-1}}$

Bipartite Graph

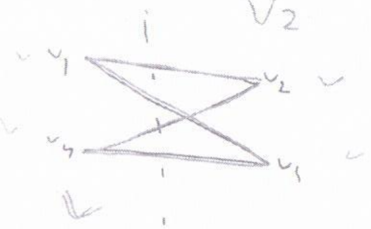
A simple graph G is called bipartite if its vertex set V can be partitioned into two disjoint sets V_1 and V_2 such that every edge in the graph connect a vertex in V_1 and a vertex in V_2 (so that no edge in G connects either two vertices in V_1 or two vertice in V_2) when this condition holds, we call the pair (V_1, V_2) a partition of the vertex set V of G.



$V = \{v_1, v_2, v_3, v_4\}$

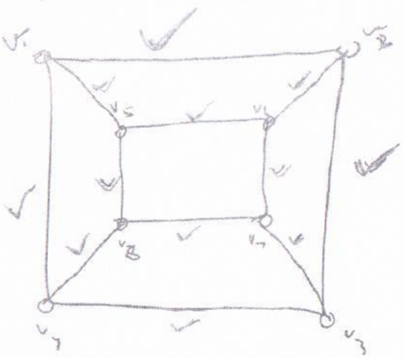
$V_1 = \{v_1, v_4\}$

$V_2 = \{v_2, v_3\}$



So it is bipartite graph

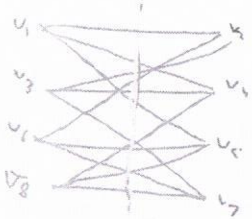
Q



$V = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8\}$

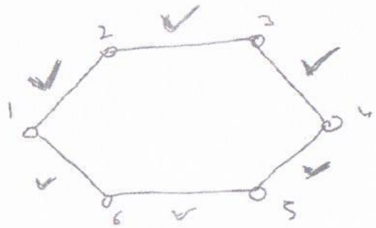
$V_1 = \{v_1, v_3, v_6, v_8\}$

$V_2 = \{v_2, v_4, v_5, v_7\}$



Q Show that $\boxed{C_6}$ is a bipartite graph.

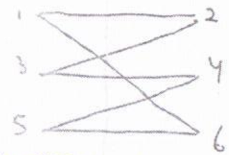
$C_6 \to$ it is cycle with 6 vertices



$V = \{1, 2, 3, 4, 5, 6\}$

$V_1 = \{1, 3, 5\}$

$V_2 = \{2, 4, 6\}$



So it is a bipartite graph