$\square$

BTECH
(SEM I) THEORY EXAMINATION 2021-22
ENGINEERING MATHEMATICS-I
Time: 3 Hours
Total Marks: 100

## Notes:

- Attempt all Sections and Assume any missing data.
- Appropriate marks are allotted to each question, answer accordingly.

| SECT | ION-A Attempt All of the following Questions in brief Marks(10X2=20) | CO |
| :---: | :---: | :---: |
| Q1(a) | If the matrix $A=\left[\begin{array}{ccc}-1 & 2 & 3 \\ 0 & 3 & 5 \\ 0 & 0 & -2\end{array}\right]$, then find the eigen value of $A^{3}+5 A+8 I$ | 1 |
| Q1(b) | Reduce the matrix $\left[\begin{array}{lll}1 & 1 & 1 \\ 3 & 1 & 1\end{array}\right]$ into normal form and find its rank. | 1 |
| Q1(c) | Find the envelope of the family of straight line $y=m x+\frac{a}{m}$, where m is a parameter. | 2 |
| Q1(d) | Can mean value theorem be applied to $f(x)=\tan x$ in $[0, \pi]$. | 2 |
| Q1(e) | State Euler's Theorem on homogeneous function. | 3 |
| Q1(f) | Find the critical points of the function $f(x, y)=x^{3}+y^{3}-3 a x y$. | 3 |
| Q1(g) | Find the area bounded by curve $y^{2}=x$ and $x^{2}=y$. |  |
| Q1(h) | Find the value of $\int_{0}^{1} \int_{0}^{x} \int_{0}^{x+y} d x d y d z$. |  |
| Q1(i) | Find a unit normal vector to the surface $z^{2}=x^{2}+y^{2}$ a | 5 |
| Q1(j) | State Stoke's Theorem. | 5 |
| SECT | ION-B Attempt ANY THREE of the following Questions Marks $\mathbf{3 X 1 0}=\mathbf{3 0}$ ) | CO |
| Q2(a) | Find the characteristic equation of the matrix $A=\left[\begin{array}{lll}2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2\end{array}\right]$, compute $A^{-1}$ and prove that $A^{8}-5 A^{7}+7 A^{6}-3 A^{5}+A^{4}-5 A^{3}+8 A^{2}-2 A+I=\left[\begin{array}{lll}8 & 5 & 5 \\ 0 & 3 & 0 \\ 5 & 5 & 8\end{array}\right]$. | 1 |
| Q2(b) | State Rolle's theorem and verify Rolle's theorem for the function $f(x)=\frac{\sin x}{e^{x}}$ in $[0, \pi]$. | 2 |
| Q2(c) | If $u, v$ and $w$ are the roots of $(\lambda-x)^{3}+(\lambda-y)^{3}+(\lambda-z)^{3}=0$, cubic in $\lambda$, find $\frac{\partial(u, v, w)}{\partial(x, y, z)}$. | 3 |
| Q2(d) | Find the volume bounded by the cylinder $x^{2}+y^{2}=4$ and the plane $y+z=4$ and $z=0$. | 4 |
| Q2(e) | Apply Green's theorem to evaluate $\int_{C}\left[\left(2 x^{2}-y^{2}\right) d x+\left(x^{2}+y^{2}\right) d y\right]$,where C is th boundary of the area enclosed by the x -axis and the upper half of the circle $x^{2}+y^{2}=a^{2} .$ | 5 |


$\square$

| SECTION-C $\quad$ Attempt ANY ONE following Question $\quad$ Marks $(\mathbf{1 X 1 0}=\mathbf{1 0})$ | CO |
| :--- | :--- | :---: |
| Q4(a) | If $f(x)=\frac{x}{1+e^{\frac{1}{x}}} ; \quad x \neq 0$ and $f(0)=0$, then show that the function is continuous |
| but not differentiable at $x=0$. | 2 |
| Q4(b) | If $y=\left(x+{\left.\sqrt{1+x^{2}}\right)^{m}, \text { find } y_{n}(0) .}^{2}\right.$ |


| SECTION-C Attempt ANY ONE following Question Marks $(\mathbf{1 X 1 0 = 1 0})$ | CO |  |
| :--- | :--- | :---: |
| Q5(a) | Expand $x^{y}$ in powers of $(x-1)$ and $(y-1)$ up to the third-degree terms and hence <br> evaluate $(1.1)^{1.02}$. | 3 |
| Q5(b) | A rectangular box which is open at the top having capacity 32c.c. Find the <br> dimension of the box such that the least material is required for its constructions. | 3 |


| SECTION-C Attempt ANY ONE following Question $\quad$ Marks (1X10=10) | CO |
| :--- | :--- | :---: |
| Q6(a) | Change the order of integration in $I=\int_{0}^{1} \int_{x^{2}}^{2-x} x y d y d x$ and hence evaluate the |
|  | same. |


| SECTION-C Attempt ANY ONE following Question $\quad$ Marks (1X10=10) | CO |  |
| :--- | :--- | :--- | :---: |
| Q7(a) | Find the directional derivative of $\nabla(\nabla f)$ at the point $(1,-2,1)$ in the direction of <br> the normal to the surface $x y^{2} z=3 x+z^{2} \quad$ where $f=2 x^{3} y^{2} z^{4}$. | 5 |
| Q7(b) | Find the constants $a, b, c$ so that <br> $\vec{F}=(x+2 y+a z) \hat{\imath}+(b x-3 y-z) \hat{\jmath}+(4 x+c y+2 z) \hat{k}$ is irrotational and hence <br> find function $\emptyset$ such that $\vec{F}=\nabla \emptyset$. | 5 |

