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BTECH
(SEM III) THEORY EXAMINATION 2022-23
MATHEMATICS-III

Time: 3 Hours ***Total Marks: 100***

Note: 1. Attempt all Sections. If require any missing data; then choose suitably.

SECTION A

1. Attempt *all* questions in brief. **2 x 10 = 20**

(a) Find $L\{t\cos at\}$.

(b) Evaluate $L^{-1}\left\{\frac{1}{s-1}+\frac{2}{s+3}+\frac{2}{s^3}\right\}$.

(c) Write the formula for Fourier integral representation of a function $f(x)$ with appropriate properties of $f(x)$.

(d) Find Fourier transform of $f(x)=\begin{cases}1, & -1\le x\le 1\\ 0, & otherwise.\end{cases}$

(e) Define a Ring.

(f) Sate Lagrange's theorem.

(g) Write the conditions for a relation to be an equivalence relation.

(h) If $f(x)=\cos(\log x)$, then prove that $f(a)f(b)-\frac{1}{2}\left\{f\left(\frac{a}{b}\right)+f(ab)\right\}=0$.

(i) Draw Hasse diagram for the POSET $(A,\le)$, where $A=\{1,2,3,9,18\}$ and relation "$\le$" is the "divides" relation.

(j) Let $A=\{2,3,4,6,8,24,48\}$ with partial ordering of divisibility. Determine all the maximal and minimal elements of A.

SECTION B

2. Attempt any *three* of the following: **10x3=30**

(a) If $L\{f(t)\}=F(s)$, then prove that
$$L\left\{\int_0^t f(u)du\right\}=\frac{1}{s}F(s).$$

(b) Find inverse Fourier sine transform of $\frac{s}{1+s^2}$.

(c) Show that the set $G=\{1,-1,i,-i\}$ of all 4^{th} roots of unity forms a multiplicative abelian group. Also find the order of each element of G.

(d) Establish the following formula by the method of mathematical induction:
$$1+3+5+\cdots+(2n-1)=n^2.$$

(e) Determine the disjunctive normal form of the following Boolean expression:
$$x\wedge(y\vee z).$$

SECTION C

3. Attempt any *one* part of the following: **10x1=10**

(a) By using the method of Laplace transform, solve the initial value problem
$$y''+2y-3y=3, y(0)=4, y'(0)=-7.$$

(b) Use convolution theorem to find inverse Laplace transform of $\frac{1}{s^2(s+1)^2}$.

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4. Attempt any *one* part of the following: 10x1=10

(a) If $f(x) = e^{-|x|}$, then prove that the Fourier transform $F\{f(x)\} = \frac{2}{1+s^2}$.

(b) Solve the following difference equation by using Z- transform:

$$y_{n+2} - 3y_{n+1} + 2y_n = 0, \qquad y_0 = -1, y_1 = 2.$$

5. Attempt any *one* part of the following: 10x1=10

(a) Show that the proposition $p \vee \sim (p \wedge q)$ is tautology.

(b) Find the congruent solutions of the following:

(i) $2x + 1 \equiv 4(mod\ 5)$,

(ii) $51x \equiv 32(mod\ 7)$.

6. Attempt any *one* part of the following: 10x1=10

(a) Find the solution for the recurrence relation:

$$\begin{cases} x_n = 2x_{n-1} - 5x_{n-2}, & n \geq 2 \\ x_0 = 1 \\ x_1 = 5 \end{cases}$$

(b) Find the coefficient of x^{2005} in the generating function $G(x) = \frac{1}{(1-x)^2(1+x)^2}$.

7. Attempt any *one* part of the following: 10x1=10

(a) Given the Boolean expression $f = ABC + B\bar{C}D + \bar{A}BC$. Then,

(i) Make a truth table,

(ii) Simplify using k −map,

(iii) Make a switching circuit of the expression.

(b) Define distributive lattices. Prove that in a distributive lattice $(L, \wedge, \vee)$,$(a \wedge b) \vee (b \wedge c) \vee (c \wedge a) = (a \vee b) \wedge (b \vee c) \wedge (c \vee a)$ holds for all $a, b, c \in L$.

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